Chapter 3

Material Balances

Material balance calculations are employed in tracing the inflow and outflow of material in a process and thus establish quantities of components or the whole process stream. The procedures are useful in formulating products to specified compositions from available raw materials, evaluating final compositions after blending, evaluating processing yields, and evaluating separation efficiencies in mechanical separation systems.

3.1 BASIC PRINCIPLES

3.1.1 Law of Conservation of Mass

Material balances are based on the principle that matter is neither created nor destroyed. Thus, in any process, a mass balance can be made as follows:

Inflow = Outflow + Accumulation

Inflow may include formation of material by chemical reaction or microbial growth processes, and outflow may include material depletion by chemical or biological reactions.

If accumulation is 0, inflow = outflow and the process is at steady state. If the accumulation term is not 0, then the quantity and concentration of components in the system could change with time and the process is an unsteady state.

3.1.2 Process Flow Diagrams

Before formulating a material balance equation, visualize the process and determine the boundary of the system for which the material balance is to be made. It is essential that everything about the process that affects the distribution of components is known. The problem statement should be adequate to enable the reader to draw a flow diagram. However, in some cases, basic physical principles associated with a process may affect the distribution of components in the system but may not be stated in the problem. It is essential that a student remembers the physical principles applied in the processes used as examples. Knowing these principles not only allows the student to solve similar material balance problems but also provides information that may be used later as a basis for the design of a new process or for evaluation of parameters affecting efficiency of a process.

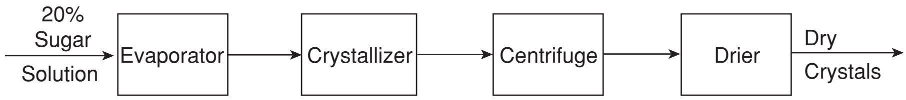

Figure 3.1 Process flow diagram for a crystallization problem.

For example, consider a problem in crystallization. The problem may be simply stated: Determine the amount of sugar (water-free basis) that can be produced from 100 kg of sugar solution that contains 20% by weight of sugar and 1% of a water-soluble uncrystallizable impurity. The solution is concentrated to 75% sugar, cooled to 20°C, centrifuged, and the crystals dried.

The problem statement is indeed adequate to draw a process flow diagram. This is shown in Fig. 3.1.

However, this flow diagram does not give a complete picture of where various streams separate and leave the system.

Figure 3.2 is a flow diagram of the same process, but after taking into consideration how components partition in various steps in the process, additional streams leaving the system are drawn in the diagram. To concentrate a 20% solution to 75% requires the removal of water. Thus, water leaves the system at the evaporator. The process of cooling does not alter the mass, therefore, the same process stream enters and leaves the crystallizer. Centrifugation separates most of the liquid phase from the solid phase, and the crystals, the solid phase containing a small amount of retained solution, enter the drier. A liquid phase leaves the system at the centrifuge. Water leaves the system at the drier.

Three physical principles involved in this problem are not stated: (1) Crystals will crystallize out of a solution when solute concentration exceeds the saturation concentration. The solute concentration in the liquid phase is forced toward the saturation concentration as crystals are formed. Given enough time to reach equilibrium, the liquid phase that leaves the system at the centrifuge is a saturated sugar solution. (2) The crystals consist of pure solute and the only impurities are those adhering to the crystals from the solution. (3) It is not possible to completely eliminate the liquid from the solid phase by centrifugation. The amount of impurities that will be retained with the sugar crystals depends on

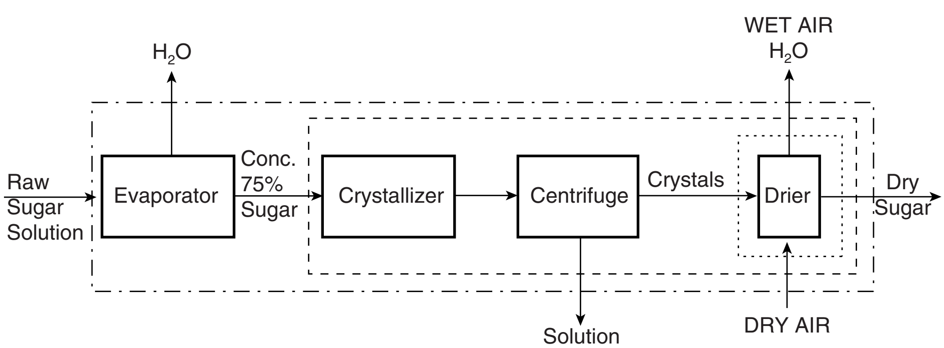

Figure 3.2 Process flow diagram for a crystallization problem showing input and exit streams and boundaries enclosing subsystems for analyzing sections of the process.

how efficiently the centrifuge separates the solid from the liquid phase. This principle of solids purity being dependent on the degree of separation of the solid from the liquid phase applies not only in crystallization but also in solvent extraction.

In order to solve this problem, the saturation concentration of sugar in water at 20°C, and the water content of the crystals fraction after centrifugation must be known.

3.1.3 System Boundaries

Figure 3.2 shows how the boundaries of the system can be moved to facilitate solving the problem. If the boundary completely encloses the whole process, there will be one stream entering and four streams leaving the system. The boundary can also be set just around the evaporator in which case there is one stream entering and two leaving. The boundary can also be set around the centrifuge or around the drier. A material balance can be carried out around any of these subsystems or around the whole system. The material balance equation may be a total mass balance or a component balance.

3.1.4 Total Mass Balance

The equation in section "Law of Conservation of Mass," when used on the total weight of each stream entering or leaving a system, represents a total mass balance. The following examples illustrate how total mass balance equations are formulated for systems and subsystems.

Example 3.1. In an evaporator, dilute material enters and concentrated material leaves the system. Water is evaporated during the process. If I is the weight of the dilute material entering the system, W is the weight of water vaporized, and C is the weight of the concentrate, write an equation that represents the total mass balance for the system. Assume that a steady state exists.

Solution:

The problem statement describes a system depicted in Fig. 3.3.
The total mass balance is

Inflow = Outflow + Accumulation

$I = W + C$ (accumulation is 0 in a steady-state system)

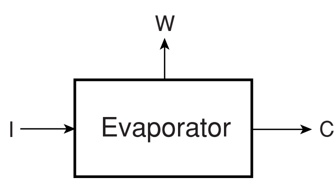

Figure 3.3 Input and exit streams in an evaporation process.

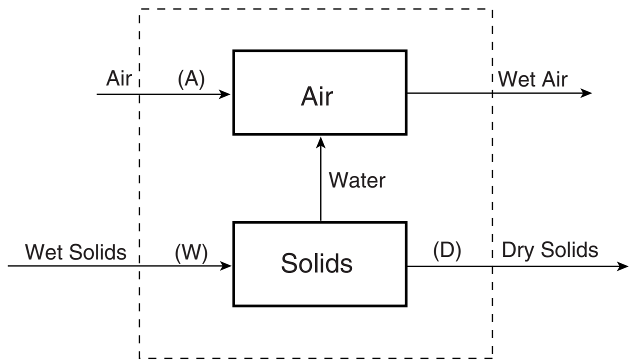

Figure 3.4 Diagram of material flow in a dehydration process.

Example 3.2. Construct a diagram and set up a total mass balance for a dehydrator. Air enters at the rate of A lb/min, and wet material enters at W lb/min. Dry material leaves the system at D lb/min. Assume steady state.

Solution:

The problem statement describes a system (dehydrator) where air and wet material enters and dry material leaves. Obviously, air must leave the system also, and water must leave the system. A characteristic of a dehydrator not written into the problem statement is that water removed from the solids is transferred to air and leaves the system with the air stream. Figure 3.4 shows the dehydrator system and its boundaries. Also shown are two separate subsystems—one for the solids and the other for air—with their corresponding boundaries. Considering the whole dehydrator system, the total mass balance is

$$W + A = \text{wet air} + D$$

Considering the air subsystem:

$$A + \text{water} = \text{wet air}$$

The mass balance for the solids subsystem is

$$W = \text{water} + D$$

Example 3.3. Orange juice concentrate is made by concentrating single-strength juice to 65% solids followed by dilution of the concentrate to 45% solids using single-strength juice. Draw a diagram for the system and set up mass balances for the whole system and for as many subsystems as possible.

Solution:

The problem statement describes a process depicted in Fig. 3.5. Consider a hypothetical proportionator that separates the original juice (S) to that which is fed to the evaporator (F) and that (A) which is used to dilute the 65% concentrate. Also, introduce a blender to indicate that part of the process where

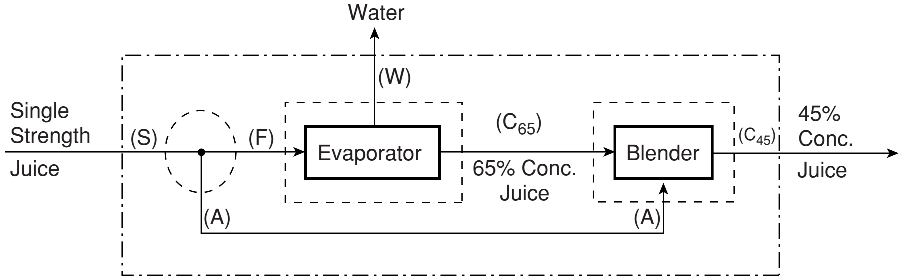

Figure 3.5 Diagram of an orange juice concentrate process involving evaporation and blending of concentrate with freshly squeezed juice.

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the 65% concentrate (C_{65}) and the single-strength juice are mixed to produce the 45% concentrate (C_{45}). The material balance equations for the whole system and the various subsystems are:

Overall: $S = W + C_{45}$
Proportionator: $S = F + A$
Evaporator: $F = W + C_{65}$
Blender: $C_{65} + A = C_{45}$

3.1.5 Component Mass Balance

The same principles apply as in the total mass balance except that components are considered individually. If there are n components, n independent equations can be formulated; one equation for total mass balance and n − 1 component balance equations.

Because the object of a material balance problem is to identify the weights and composition of various streams entering and leaving a system, it is often necessary to establish several equations and simultaneously solve these equations to evaluate the unknowns. It is helpful to include the known quantities of process streams and concentrations of components in the process diagram in order that all streams where a component may be present can be easily accounted for. In a material balance, use mass units and concentration in mass fraction or mass percentage. If the quantities are expressed in volume units, convert to mass units using density.

A form of a component balance equation that is particularly useful in problems involving concentration or dilution is the expression for the mass fraction or weight percentage.

$$\text{Mass fraction A} = \frac{\text{mass of component A}}{\text{total mass of mixture containing A}}$$

Rearrange the equation:

$$\text{Total mass of mixture containing A} = \frac{\text{mass of component A}}{\text{mass fraction of A}}$$

Thus, if the weight of component A in a mixture is known, and its mass fraction in that mixture is known, the mass of the mixture can be easily calculated.

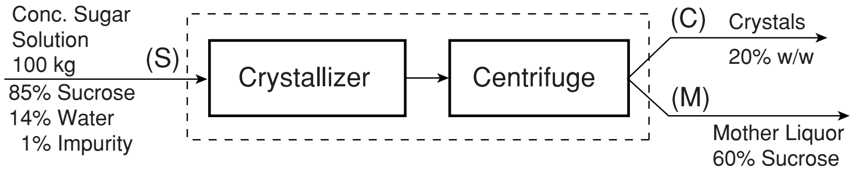


Figure 3.6 Diagram showing composition and material flow in a crystallization process.

Example 3.4. Draw a diagram and set up a total mass and component balance equation for a crystallizer where 100 kg of a concentrated sugar solution containing 85% sucrose and 1% inert, water-soluble impurities (balance, water) enters. Upon cooling, the sugar crystallizes from solution. A centrifuge then separates the crystals from a liquid fraction, called the mother liquor. The crystal slurry fraction has, for 20% of its weight, a liquid having the same composition as the mother liquor. The mother liquor contains 60% sucrose by weight.

Solution:

The diagram for the process is shown in Fig. 3.6. Based on a system boundary enclosing the whole process of crystallization and centrifugation, the material balance equations are as follows.

Total mass balance:

$$S = C + M$$

Component balance on sucrose:

$$S(0.85) = M(0.6) + C(0.2)(0.6) + C(0.8)$$

The term on the left is sucrose in the inlet stream. The first term on the right is sucrose n the mother liquor. The second term on the right is sucrose in mother liquor carried by crystals. The last term on the right is sucrose in the crystals.

Component balance on water:

Let x = mass fraction of impurity in the mother liquor

$$S(0.14) = M(0.4 - x) + C(0.2)(0.4 - x)$$

The first term on the left is water in the inlet stream. The first term on the right is water in the mother liquor. The last term on the right is water in the mother liquor adhering to the crystals.

Component balance on impurity:

$$S(0.01) = M(x) + C(0.2)(x)$$

Note that a total of four equations can be formulated but there are only three unknown quantities (C, M, and x). One of the equations is redundant.

Example 3.5. Draw a diagram and set up equations representing total mass balance and component mass balance for a system involving the mixing of pork (15% protein, 20% fat, and 63% water) and backfat (15% water, 80% fat, and 3% protein) to make 100 kg of a mixture containing 25% fat.

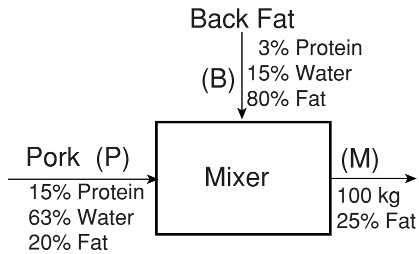


Figure 3.7 Composition and material flow in a blending process.

Solution:

The diagram is shown in Fig. 3.7.
Total mass balance:

$$P + B = 100$$

Fat balance:

$$0.2\ P + 0.8\ B = 0.25(100)$$

These two equations are solved simultaneously by substituting $P = 100 - B$ into the second equation.

$$0.2(100 - B) + 0.8\ B = 25$$

$$B = \frac{25 - 20}{0.8 - 0.2} = 8.33\ \text{kg}$$

$$P = 100 - 8.33 = 91.67\ \text{kg}$$

3.1.6 Basis and "Tie Material"

A "tie material" is a component used to relate the quantity of one process stream to another. It is usually the component that does not change during a process. Examples of tie material are solids in dehydration or evaporation processes and nitrogen in combustion processes. Although it is not essential that these tie materials are identified, the calculations are often simplified if it is identified and included in one of the component balance equations. This is illustrated in Example 3.6 of the section "A Steady State" where the problem is solved rather readily using a component mass balance in the solid (the tie material in this system) compared to when the mass balance was made on water. In a number of cases, the tie material need not be identified as illustrated in examples in the section "Blending of Food Ingredients."

A "basis" is useful in problems where no initial quantities are given and the answer required is a ratio or a percentage. It is also useful in continuous flow systems. Material balance in a continuous flow system is done by assuming as a basis a fixed time of operation. A material balance problem can be solved on any assumed basis. After all the quantities of process streams are identified, the specific quantity asked in the problem can be solved using ratio and proportion. It is possible to change basis when considering each subsystem within a defined boundary inside the total system.

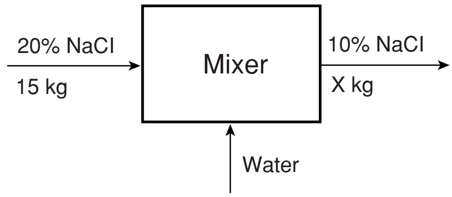


Figure 3.8 Composition and material flow for a dilution process.

3.2 MATERIAL BALANCE PROBLEMS INVOLVED IN DILUTION, CONCENTRATION, AND DEHYDRATION

3.2.1 Steady State

These problems can be solved by formulating total mass and component balance equations and solving the equations simultaneously.

Example 3.6. How many kilograms of a solution containing 10% NaCl can be obtained by diluting 15 kg of a 20% solution with water?

Solution:

The process diagram in Fig. 3.8 shows that all NaCl enters the mixer with the 20% NaCl solution and leaves in the diluted solution. Let x = kg 10% NaCl solution; y = kg water. The material balance equations are

Total mass: $15 = X - Y$
Component: $15(0.20) = X(0.10)$

The total mass balance equation is redundant because the component balance equation alone can be used to solve the problem.

$$x = \frac{3}{0.1} = 30 \text{ kg}$$

The mass fraction equation can also be used in this problem. Fifteen kilograms of a 20% NaCl solution contains 3 kg NaCl. Dilution would not change the quantity of NaCl so that 3 kg of NaCl is in the diluted mixture. The diluted mixture contains 10% NaCl, therefore:

$$x = \frac{3\text{ kg NaCl}}{\text{mass fraction NaCl}} = \frac{3}{0.1} = 30 \text{ kg}$$

Example 3.7. How much weight reduction would result when a material is dried from 80% moisture to 50% moisture?

Solution:

The process diagram is shown in Fig. 3.9. Dehydration involves removal of water and the mass of solids remain constant. There are two components, solids and water, and a decrease in the concentration

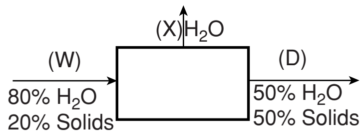


Figure 3.9 Composition and material flow in a dehydration process.

of water, indicating a loss, will increase the solids concentration. Let: W = mass of 80% moisture material, D = mass of 50% moisture material, and X = mass water lost, which is also the reduction in mass.

No weights are specified, therefore express reduction in weight as a ratio of final to initial weight. The material balance equations are

Total mass: $W = X + D$
Water: $0.8W = 0.5\,D + X$

Solving simultaneously:

$$W = X + D; X = W - D$$
$$0.8\,W = 0.5\,D + (W - D)$$
$$0.5\,D = 0.2\,W;$$
$$\frac{D}{W} = \frac{0.2}{0.5} = 0.4$$
$$\%\text{wt. reduction} = \frac{W - D}{W}(100) = \left(1 - \frac{D}{W}\right)100$$
$$= (1 - 0.4)(100) = 60\%$$

The problem can also be solved by using as a basis 100 kg of 80% moisture material. Let W = 100.

Total mass balance: $100 = X + D$
Water balance: $0.8(100) = X + 0.5\,D$

Solving simultaneously, by subtracting one equation from the other:

$$100 - 80 = D(1 - 0.5);\ D = 20/0.5 = 40\text{ kg}.$$
$$\%\text{ wt. reduction} = \frac{100 - 40}{100}(100) = 60\%$$

3.2.2 Volume Changes on Mixing

When two liquids are mixed, the volumes are not always additive. This is true with most solutions and miscible liquids. Sodium chloride solution, sugar solutions, and ethanol solution all exhibit volume changes on mixing. Because of volume changes, material balances must be done on mass rather than volume of components. Concentrations on a volume basis must be converted to a mass basis before the material balance equations are formulated.

Example 3.8. Alcohol content in beverages are reported as percent by volume. A "proof" is twice the volume percent of alcohol. The density of absolute ethanol is 0.7893 g/cm^3. The density of a solution containing 60% by weight of ethanol is 0.8911 g/cm^3. Calculate the volume of absolute ethanol that must be diluted with water to produce 1 liter of 60% by weight, ethanol solution. Calculate the "proof" of a 60% ethanol solution.

Solution:

Use as a basis: 1 liter of 60% w/w ethanol.
Let X = volume of absolute ethanol in liters. The component balance equation on ethanol is:

$$X(1000)(0.7983) = 1(1000)(0.8911)(0.6)$$
$$X = 0.677 \text{ L}$$
$$\text{g water} = 1000(0.8911) - 0.677(0.7983) = 356.7 \text{ g or } 356.7 \text{ mL}$$

Total volume components = 0.677 + 0.3567 = 1.033 L

This is a dilution problem, and a component balance on ethanol was adequate to solve the problem. Note that a mass balance was made using the densities given. There is a volume loss on mixing as more than 1 liter of absolute ethanol and water produced 1 liter of 60% w/w ethanol solution.

To calculate the "proof" of 60% w/w ethanol, use as a basis 100 g of solution.

Volume of solution = 100/0.8911 = 112.22 cm^3
Volume of ethanol = 100(0.6)/0.7893 = 76.016 cm^3
Volume percent = (76.016/112.22)(100) = 67.74%
Proof = 2 (volume percent) = 135.5 proof

3.2.3 Continuous Versus Batch

Material balance calculations are the same regardless whether a batch or continuous process is being evaluated. In a batch system, the total mass considered includes what entered or left the system at one time. In a continuous system, a basis of a unit time of operation may be used and the material balance will be made on what entered or left the system during that period of time. The previous examples were batch operations. If the process is continuous, the quantities given will all be mass/time (e.g., kg/h). If the basis used is 1 hour of operation, the problem is reduced to the same form as a batch process.

Example 3.9. An evaporator has a rated evaporation capacity of 500 kg water/h. Calculate the rate of production of juice concentrate containing 45% total solids from raw juice containing 12% solids.

Solution:

The diagram of the process is shown in Fig. 3.10. Use as a basis 1 hour of operation. Five hundred kg of water leaves the system. A component balance on solids and a total mass balance will be needed to solve the problem. Let F = the feed, 12% solids juice, and C = concentrate containing 45% solids. The material balance equations are:

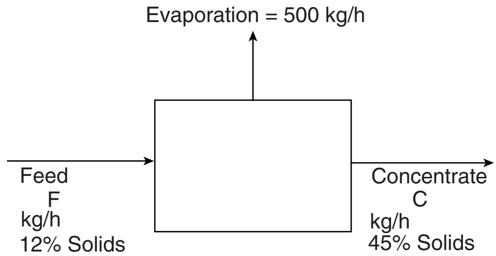


Figure 3.10 Flow diagram of an evaporation process.

Total mass:

$F = C + 500$

Solids:

$$0.12\,F = 0.45\,C;\ F = \frac{0.45\,C}{0.12} = 3.75\,C$$

$$C = \frac{500}{3.75 - 1} = 181.8\ \text{kg}$$

Substituting and solving for C: $3.75\ C = C + 500$.

Because the basis is 1 hour of operation, the answer will be: Rate of production of concentrate = 181.8 kg/h

3.2.4 Recycle

Recycle is evaluated similarly as in the previous examples, but the boundaries of subsystems analyzed are moved around to isolate the process streams being evaluated. A system is defined that has a boundary surrounding the recycle stream. If this total system is analyzed, the problem may be reduced to a simple material balance problem without recycle.

Example 3.10. A pilot plant model of a falling film evaporator has an evaporation capacity of 10 kg water/h. The system consists of a heater through which the fluid flows down in a thin film and the heated fluid discharges into a collecting vessel maintained under a vacuum where flash evaporation reduces the temperature of the heated fluid to the boiling point. In continuous operation, a recirculating pump draws part of the concentrate from the reservoir, mixes this concentrate with feed, and pumps the mixture through the heater. The recirculating pump moves 20 kg of fluid/h. The fluid in the collecting vessel should be at the desired concentration for withdrawal from the evaporator at any time. If feed enters at 5.5% solids and a 25% concentrate is desired, calculate: (a) the feed rate and concentrate production rate, (b) the amount of concentrate recycled, and (c) concentration of mixture of feed and recycled concentrate.

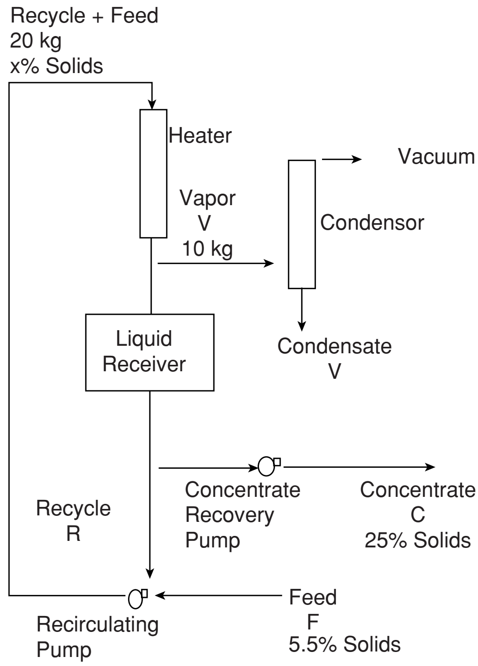


Figure 3.11 Diagram of material flow in a falling film evaporator with product recycle.

Solution:

A diagram of the system is shown in Fig. 3.11. The basis is 1 hour of operation.

A mass and solids balance over the whole system will establish the quantity of feed and concentrate produced per hour.

Total mass:

$$F = C + V;\ F = C + 10$$

Solids:

$$F(0.055) = C(0.25);\ F = C\left(\frac{0.25}{0.055}\right) = 4.545\,C$$

Substituting F:

$$4.545\,C = C + 10;\ C = \frac{10}{4.545 - 1} = 2.82\ \text{kg}$$

(a) Solving for F : F = 4.545(2.82) = 12.82 kg/h. The concentrate production rate is 2.82 kg/h.

(b) Material balance around the recirculating pump:

$$R + F = 20;\ R = 20 - 12.82 = 7.18\ \text{kg}$$

$$\text{Recycle rate} = 7.18\ \text{kg/h}$$

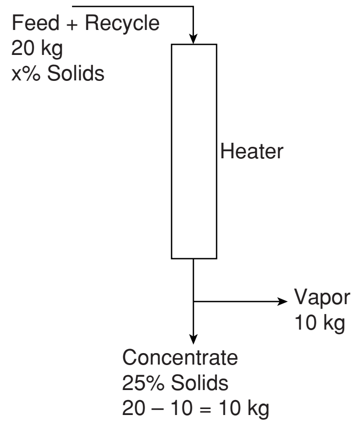


Figure 3.12 Diagram of material balance around the heater of a falling film evaporator.

(c) A material balance can be made around the part of the system where the vapor separates from the heated fluid as shown in Fig. 3.12.

$$\text{Solids balance: } 20(x) = 10(0.25);\ x = 2.5/20 = 0.125.$$

The fluid entering the heater contains 12.5% solids.

3.2.5 Unsteady State

Unsteady-state material balance equations involve an accumulation term in the equation. Accumulation is expressed as a differential term of the rate of change of a variable with respect to time. A mass balance is made in the same manner as steady-state problems. Because a differential term is involved, the differential equation must be integrated to obtain an equation for the value of the dependent variable as a function of time.

Example 3.11. A stirred tank with a volume of 10 liters contains a salt solution at a concentration of 100 g/L. If salt-free water is continuously fed into this tank at the rate of 12 L/h, and the volume is maintained constant by continuously overflowing the excess fluid, what will be the concentration of salt after 90 min.

Solution:

This is similar to a dilution problem, except that the continuous overflow removes salt from the tank, thus reducing not only the concentration but also the quantity of salt present. A mass balance will be made on the mass of salt in the tank and in the streams entering and leaving the system.

Let x = the concentration of salt in the vessel at any time. Representing time by the symbol t, the accumulation term will be dx/dt, which will have units of g salt/(L $\cong$ min). Convert all time units to minutes in order to be consistent with the units. Multiplying the differential term by the volume will result in units of g salt/min. The feed contains no salt, therefore the input term is zero. The overflow is at the same rate as the feed, and the concentration in the overflow is the same as inside the vessel, therefore the output term is the feed rate multiplied by the concentration inside the vessel. The mass balance equation is

$$0 = Fx + V\left(\frac{dx}{dt}\right); \frac{dx}{dt} = -\frac{F}{V}x$$

The negative sign indicates that x will be decreasing with time. Separating variables:

$$\frac{dx}{x} = -\frac{F}{V}dt$$

Integrating:

$$\ln x = -\left(\frac{F}{V}\right)t + C$$

The constant of integration is obtained by substituting $x = 100$ at $t = 0$; $C = \ln(100)$; $F = 12$ L/h $=$ 0.2 L/min; $V = 10$ L

$$\ln\left(\frac{x}{100}\right) = -\frac{0.2\,t}{10}$$

At 90 min: $\ln(0.01\,x) = -1.800$.

$$x = 100(e^{-1.80}) = 16.53 \text{ g/L}$$

Example 3.12. The generation time is the time required for cell mass to double. The generation time of yeast in a culture broth has been determined from turbidimetric measurements to be 1.5 hours. (a) If this yeast is used in a continuous fermentor, which is a well-stirred vessel having a volume of 1.5 L, and the inoculum is 10,000 cells/mL, at what rate can cell-free substrate be fed into this fermentor in order that the yeast cell concentration will remain constant? The fermentor volume is maintained constant by continuous overflow. (b) If the feed rate is 80% of what is needed for a steady-state cell mass, calculate the cell mass after 10 hours of operation.

Solution:

This problem is a combination of dilution with continuous cell removal, but with the added factor of cell generation by growth inside the vessel. The diagram shown in Fig. 3.13 represents the material balance on cell mass around a fermentor. The balance on cell mass or cell numbers is as follows:

Input(from feed + cell growth) = output + accumulation

(a) The substrate entering the fermentor is cell free, therefore this term in the material balance equation is zero. Let R = substrate feed rate = overflow rate, because fluid volume in fermentor is maintained constant. Let x = cells/mL. The material balance on cell numbers with the appropriate time derivatives for rate of increase in cell numbers, $(dx/dt)_{gen}$, and accumulation $(dx/dt)_{acc}$ is

$$V\left[\frac{dx}{dt}\right]_{gen} = Rx + V\left[\frac{dx}{dt}\right]_{acc}$$

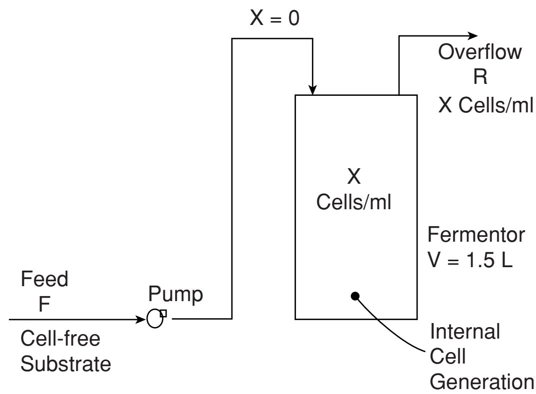


Figure 3.13 Material balance around a fermentor.

If cell mass is constant, then the last term on the right is zero. Cell growth is often expressed in terms of a generation time, g, which is the average time for doubling of cell numbers. Let t = time in hours. Then:

$$x = x_o(2)^{(t/g)}$$

Differentiating to obtain the generation term in the material balance equation:

$$\left[\frac{dx}{dt}\right]_{gen} = \frac{d}{dt}\left[x_0(2)^{(t/g)}\right] = x_0(g^{-1})(2)^{(t/g)}\ln 2 = xg^{-1}\ln 2$$

Substituting in the material balance equation and dropping out zero terms:

$$V \cdot x \cdot g^{-1}\ln 2 = R\,x;\ \text{x cancels out on both sides.}$$
$$R = V(\ln 2)g^{-1}$$

Substituting known quantities:

$$R = \frac{1.5\ln 2}{1.5} = \frac{0.693\ L}{h}$$

In continuous fermentation, the ratio R/V is the dilution rate, and when the fermentor is at a steady state in terms of cell mass, the dilution rate must equal the specific growth rate. The specific growth rate is the quotient: rate of growth/cell mass $= (1/x)(dx/dt)_{gen} = \ln 2/g$. Thus, the dilution rate to achieve steady state is strictly a function of the rate of growth of the organism and is independent of the cell mass present in the fermentor.

(b) When the feed rate is reduced, the cell number will be in an unsteady state. The material balance will include the accumulation term. Substituting the expression for cell generation into the original material balance equation with accumulation:

$$V\frac{x\ln 2}{g} = Rx + V\frac{dx}{dt}$$

$$x\left[\frac{V\ln 2}{g} - R\right] = v\frac{dx}{dt}$$

Separating variables and integrating:

$$\frac{dx}{x} = \left[\frac{\ln 2}{g} - \frac{R}{V}\right]dt$$

$$\ln\ x = \left[\frac{\ln 2}{g} - \frac{R}{V}\right]t + C$$

At $t = 0$, $x = 10,000$ cells/mL; $C = \ln(10,000)$.

$$\ln\left(\frac{x}{10,000}\right) = \left[\frac{\ln 2}{g} - \frac{R}{V}\right]t$$

Substituting: $V = 1.5$ L; $R = 0.8(0.693\ \text{L/h}) = 0.554$ L/h; $g = 1.5$ h; $t = 10$ h

$$\ln\left(\frac{x}{10,000}\right) = 0.927;\ x = 10,000(2.5269)$$

$$x = 25,269\ \text{cells/ml}$$

3.3 BLENDING OF FOOD INGREDIENTS

3.3.1 Total Mass and Component Balances

These problems involve setting up total mass and component balances and involve simultaneously solving several equations.

Example 3.13. Determine the amount of a juice concentrate containing 65% solids and single-strength juice containing 15% solids that must be mixed to produce 100 kg of a concentrate containing 45% solids.

Solution:

The diagram for the process is shown in Fig. 3.14.

Total mass balance: $X + Y = 100$; $X = 100 - Y$
Solid balance: $0.65X + 0.15Y = 100(0.45) = 45$
Substituting $(100 - Y)$ for X:

$$0.65(100 - Y) + 0.15Y = 45$$

$$65 - 0.65Y + 0.15Y = 45$$

$$65 - 45 = 0.65Y - 0.15Y$$

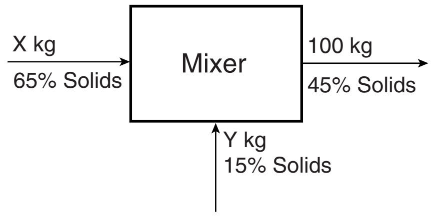


Figure 3.14 Material flow and composition in a process for blending of juice concentrates.

$20 = 0.5Y$

$Y = 40$ kg single strength juice

$X = 60$ kg 65% concentrate

Example 3.14. Determine the amounts of lean beef, pork fat, and water that must be used to make 100 kg of a frankfurter formulation. The compositions of the raw materials and the formulations are

Lean beef: 14% fat, 67% water, 19% protein.
Pork fat: 89% fat, 8% water, 3% protein.
Frankfurter: 20% fat, 15% protein, 65% water.

Solution:

The diagram representing the various mixtures being blended is shown in Fig. 3.15.

Total mass balance: $Z + X + Y = 100$
Fat balance: $0.14X + 0.89Y = 20$
Protein balance: $0.19X + 0.03Y = 15$

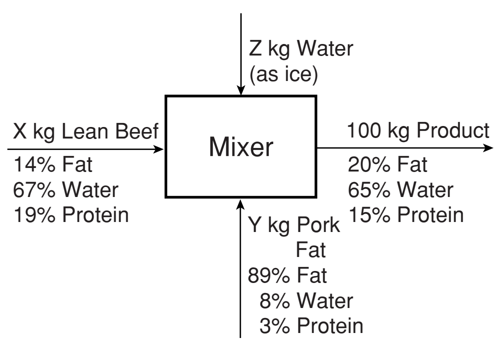


Figure 3.15 Composition and material flow in blending of meats for a frankfurter formulation.

These equations will be solved by determinants. Note that the fat and protein balance equations involve only X and Y, therefore solving these two equations first will give values for X and Y. Z will be solved using the total mass balance equation. See Section 1.9.3 for a discussion of how to set up the determinants.

The matrices for the fat and protein balance equations are

$$\begin{bmatrix} 0.14 & 0.89 \\ 0.19 & 0.03 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$$

$$X = \left| \frac{\begin{matrix} 20 & 0.89 \\ 15 & 0.03 \end{matrix}}{\begin{matrix} 0.14 & 0.89 \\ 0.19 & 0.03 \end{matrix}} \right| = \frac{(20)(0.03) - (15)(0.89)}{(0.14)(0.03) - (0.19)(0.89)}$$

$$X = \frac{-12.75}{-.1649} = 77.31$$

$$Y = \left| \frac{\begin{matrix} 0.14 & 20 \\ 0.19 & 14 \end{matrix}}{\begin{matrix} 0.14 & 0.89 \\ 0.19 & 0.03 \end{matrix}} \right| = \frac{(0.14)(14) - (0.19)(0.20)}{(0.14)(0.03) - (0.19)(0.89)}$$

$$Y = \frac{-1.7}{-0.1649} = 10.3 \text{ kg}$$

Total mass balance: $Z = 100 - 77.3 - 10.3 = 12.4$ kg.

The solution using the Solver Macro in Excel is shown in Fig. 3.16.

Example 3.15. A food mix is to be made that would balance the amount of methionine (MET), a limiting amino acid in terms of food protein nutritional value, by blending several types of plant proteins. Corn, which contains 15% protein, has 1.2 g MET/100 g protein; soy flour with 55% protein has 1.7 g MET/100 g protein; and nonfat dry milk with 36% protein has 3.2 g MET/100 g protein. How much of these ingredients must be used to produce 100 kg of formula that contains 30% protein and 2.2 g MET/100 g protein.

Solution:

This will be solved by setting up a component balance on protein and methionine. Let C = kg corn, S = kg soy flour, and M = kg nonfat dry milk. The material balance equations are

Total mass: $C + S + M = 100$

Protein: $0.15C + 0.55S + 0.36\,M = 30$

$$\text{MET: } \frac{(1.2)(0.15)}{100}C + \frac{(1.7)(0.55)}{100}S + \frac{(3.2)(0.36)}{100}M = \frac{2.2}{100}(30)$$

$$0.18C + 0.935\,S + 1.152\,M = 66$$

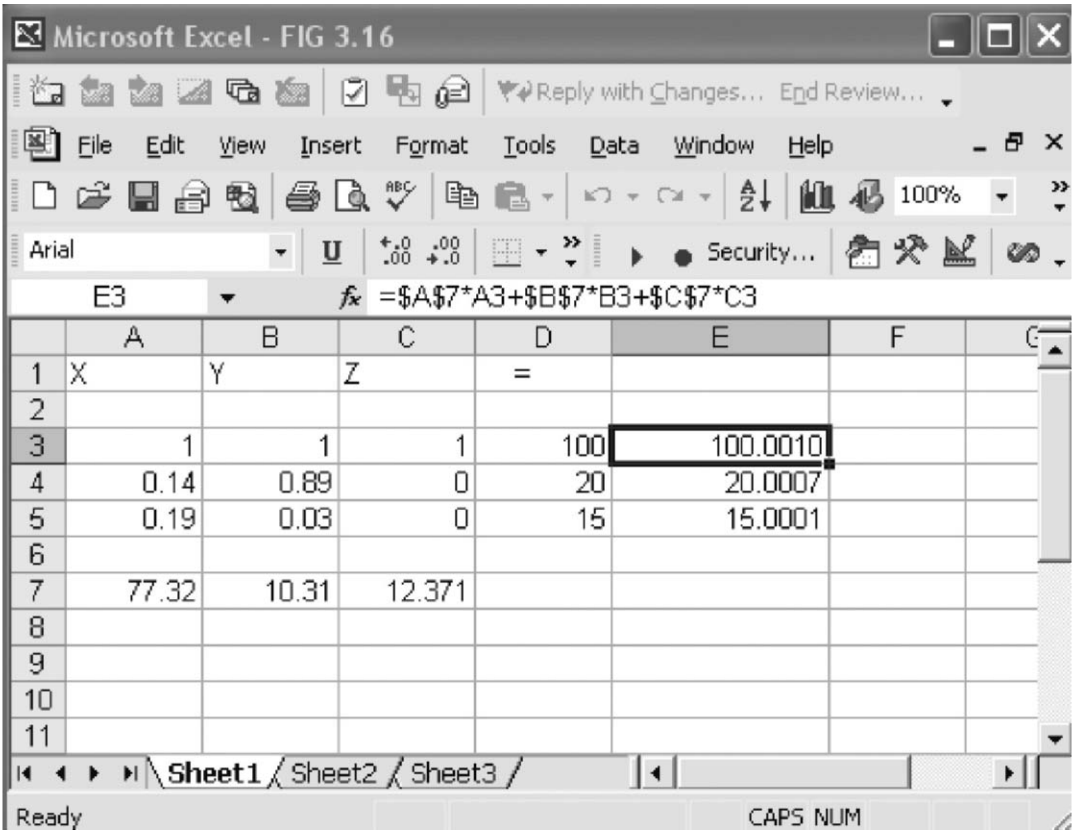


Figure 3.16 Solution to meats blending problem for frankfurter formulation using the Solver macro in Excel.

The matrices representing the three simultaneous equations are

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.15 & 0.55 & 0.36 \\ 0.18 & 0.935 & 1.152 \end{bmatrix} \begin{bmatrix} \mathrm{C} \\ \mathrm{S} \\ \mathrm{M} \end{bmatrix} = \begin{bmatrix} 100 \\ 30 \\ 66 \end{bmatrix}$$

Thus:

$$\mathrm{C} = \frac{\begin{bmatrix} 100 & 1 & 1 \\ 30 & 0.55 & 0.36 \\ 66 & 0.935 & 1.152 \end{bmatrix}}{\begin{bmatrix} 1 & 1 & 1 \\ 0.15 & 0.55 & 0.36 \\ 0.18 & 0.935 & 1.152 \end{bmatrix}}$$

The denominator matrix is resolved to:

$$1\begin{bmatrix} 0.55 & 0.36 \\ 0.935 & 1.152 \end{bmatrix} - 0.15\begin{bmatrix} 1 & 1 \\ 0.935 & 1.152 \end{bmatrix} + 0.18\begin{bmatrix} 1 & 1 \\ 0.55 & 0.36 \end{bmatrix}$$

$$= 1[0.55(1.152) - 0.935(0.36)] - 0.15[1(1.152) - 1(0.935)] + 0.18[(1)(0.36) - (0.55)(1)]$$

$$= 0.297 - 0.03255 - 0.0342 = 0.23025$$

The numerator matrix resolves as follows:

$$100\begin{bmatrix} 0.55 & 0.36 \\ 0.935 & 1.152 \end{bmatrix} - 30\begin{bmatrix} 1 & 1 \\ 0.935 & 1.152 \end{bmatrix} + 66\begin{bmatrix} 1 & 1 \\ 0.55 & 0.36 \end{bmatrix}$$

$$= 100[0.55(1.152) - 0.935(0.36)] - 30[1(1.152) - 0.935(1)] + 66[1(0.36) - 0.55(1)]$$

$$= 100(0.297) - 30(0.217) + 66(-0.19) = 10.65$$

$$C = \frac{10.65}{0.23025} = 46.25 \text{ kg}$$

$$S = \frac{\begin{bmatrix} 1 & 100 & 1 \\ 0.15 & 30 & 0.36 \\ 0.18 & 66 & 1.152 \end{bmatrix}}{0.23025}$$

The numerator matrix resolves as follows:

$$1\begin{bmatrix} 30 & 0.36 \\ 66 & 1.152 \end{bmatrix} - 0.15\begin{bmatrix} 100 & 1 \\ 66 & 1.152 \end{bmatrix} + 0.18\begin{bmatrix} 100 & 1 \\ 30 & 0.36 \end{bmatrix}$$

$$= 1[30(1.152) - 66(0.36)] - 0.15[100(1.152) - 66(1) + 0.18[100(.36) - 30(1)]$$

$$= 1(10.9) - 0.15(49.2) + 0.18(6) = 4.5$$

$$S = \frac{4.5}{0.23025} = 19.54 \text{ kg}$$

The total mass balance may be used to solve for M, but as a means of checking the calculations the matrix for M will be formulated and resolved.

$$M = \frac{\begin{bmatrix} 1 & 1 & 100 \\ 0.15 & 0.55 & 30 \\ 0.18 & 0.935 & 66 \end{bmatrix}}{0.23025}$$

The numerator matrix is resolved as follows:

$$1\begin{bmatrix} 0.55 & 30 \\ 0.935 & 66 \end{bmatrix} - 0.15\begin{bmatrix} 1 & 100 \\ 0.935 & 66 \end{bmatrix} + 0.18\begin{bmatrix} 1 & 100 \\ 0.55 & 30 \end{bmatrix}$$

$$= 1[0.55(66) - 0.935(30)] - 0.15[1(66) - 0.935(100)] + 0.18[1(30) - 0.55(100)]$$

$$M = \frac{7.875}{0.23025} = 34.2 \text{ kg} = 1(8.25) - 0.15(-27.5) + 0.18(-25) = 7.875$$

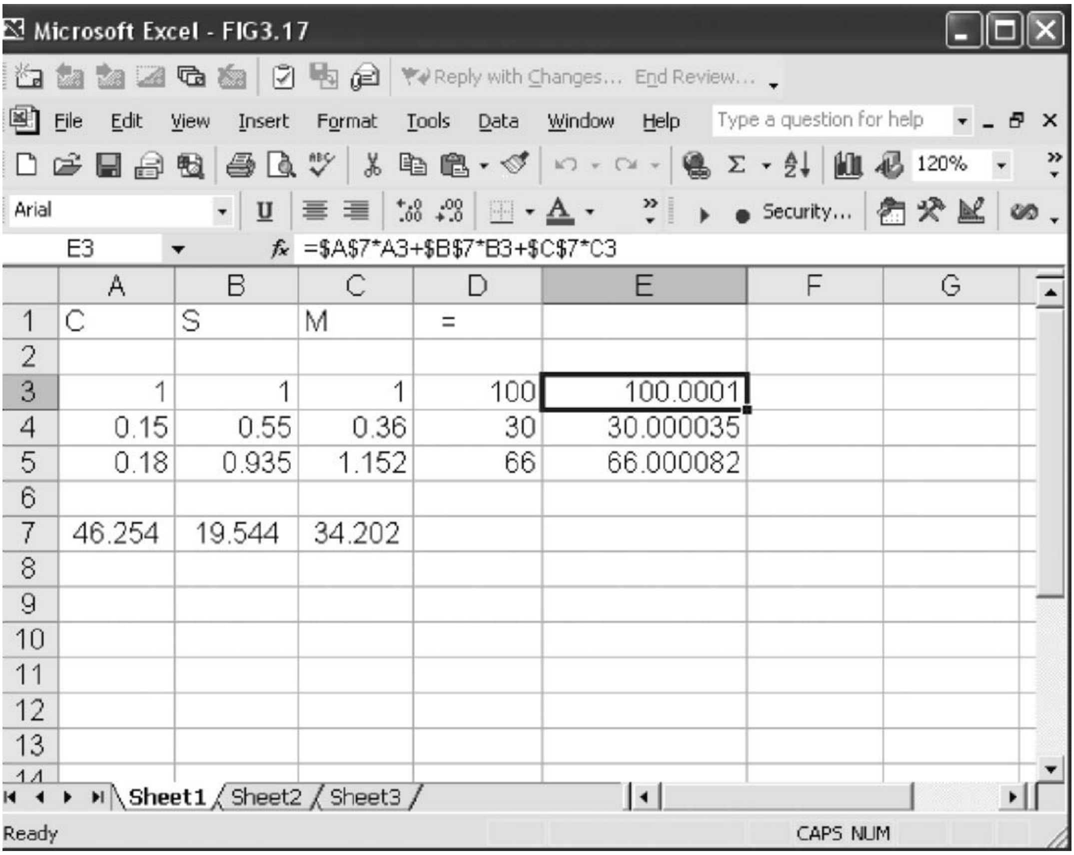


Figure 3.17 Solution to food protein blending problem using the Solver macro in Excel.

Check: C + S + M = 46.25 + 19.54 + 34.2 = 99.99.
The solution using the Solver Macro in Excel is shown in Fig. 3.17.

3.3.2 Use of Specified Constraints in Equations

When blending ingredients, constraints may be imposed either by the functional properties of the ingredients or its components, or by regulations. Examples of constraints are limits on the amount of non-meat proteins that can be used in meat products like frankfurters, moisture content in meat products, or functional properties such as fat or water binding properties. Another constraint may be cost. It is necessary that these constraints specified be included as one of the equations to be resolved.

In some cases, a number of ingredients may be available for making a food formulation. To effectively solve the quantity of each ingredient using the procedures discussed in the previous sections, the same number of independent equations must be formulated as there are unknown quantities to be solved. If there are three components that can be used to formulate component balance equations, only three independent equations can be formulated. Thus, a unique solution for components of a blend, using the solution to simultaneous equations, will be possible only if there are three ingredients to be used.

When there are more ingredients possible for use in a formulation than there are independent equations that can be formulated using the total mass and component balances, constraints will have to be used to allow consideration of all possible ingredients. The primary objective is minimizing cost and maximizing quality. Both cost and quality factors will form the basis for specifying constraints.

The constraints may not be specific (i.e., they may only define a boundary rather than a specific value). Thus, a constraint cannot be used as a basis for an equation that must be solved simultaneously with the others.

One example of a system of constraints to include as many ingredients as possible into a formulation is the "least cost formulation" concept used in the meat industry. Several software companies market "least cost formulation" strategies, and the actual algorithm within each of these computer programs may vary from one company to the other. However, the basis for these calculations is basically the same: (a) The composition must be met, usually 30% fat, and water and protein must satisfy the USDA requirement of water not to exceed 4 times the protein content plus 10% (4P + 10), or some other specified moisture range allowable for certain category of products. (b) The bind values for fat must satisfy the needs for emulsifying the fat contained within the formulation. Software companies who market least-cost sausage formulation strategies vary in how they use these fat holding capacities. (c) Water holding capacity must be maximized; that is, the water and protein content relationship should stay within the 4P + 10 allowed in the finished product. This requirement is not usually followed because of the absence of reliable water holding capacity data, and processors simply add an excess of water during formulation to compensate for the water loss that occurs during cooking.

To illustrate these concepts, a simple least-cost formulation will be set up for frankfurters utilizing a choice of five different meat types. Note that, as in the previous examples, a total mass balance and only two component balance equations can be formulated (water can be determined from the total mass balance). Thus, only three independent equations can be formulated. Product textural properties may be used as a basis for a constraint. If previous experience has shown that a minimum of the protein present in the final blend must come from a type of meat (e.g., pork) to achieve a characteristic flavor and texture, then this constraint will be one factor to be included in the analysis. Other constraints are that no negative values of the components are acceptable.

The solution will be obtained using the Solver macro in Microsoft Excel (see Chapter 1, "Simultaneous Equations").

Example 3.16. Derive a least-cost formulation involving four meat types from the five types shown. The composition and pertinent functional data, as bind values, for each of the ingredients are shown in Table 3.1. The fat content in the blend is 30% and protein is 15%.

The constraints are (1) for product color and textural considerations, at least 20% of the total protein must come from pork trim and pork cheeks; (2) the amount of beef cheek meat should not exceed 15% of the total mixture; and (3) the bind value (sum of the product of mass of each ingredient and its bind constant) must be greater than 15. These constraints are established by experience as the factors needed to impart the desired texture and flavor in the product.

Solution:

Enter data from Table 3.1 in the spreadsheet in block A5 : G10 as shown in Fig. 3.18. Designate column block H6 : H10 to hold values of the weights of the selected meat components. Designate row block B12 : H12 to hold the constraints. Enter formulas for the total mass balance in H12 and

Table 3.1 Data for the Least-Cost Formulation Problems

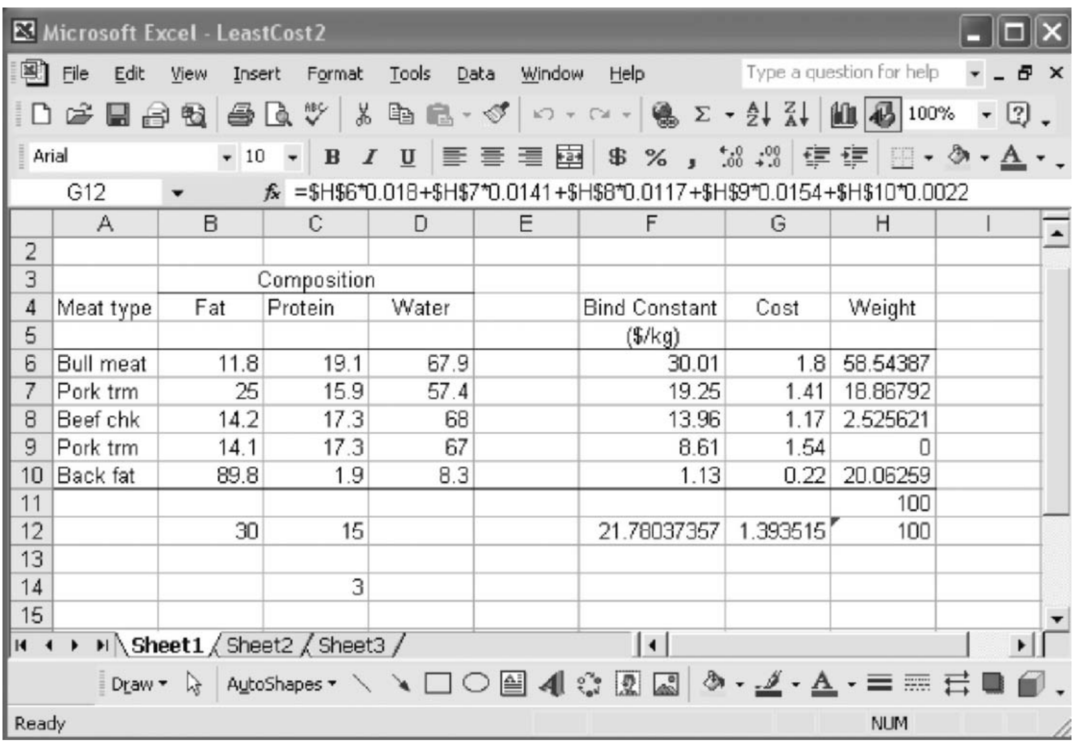
	Composition				
Meat type	*Fat*	*Protein*	*Water*	*Bind constant kg fat/100 kg*	*Cost ($/kg)*
Bull meat	11.8	19.1	67.9	30.01	1.80
Pork trm.	25.0	15.9	57.4	19.25	1.41
Beef chk.	14.2	17.3	68.0	13.96	1.17
Pork chk.	14.1	17.3	67.0	8.61	1.54
Back fat	89.8	1.9	8.3	1.13	0.22

component mass balances on fat and protein respectively in B12 and C12. Designate F12 for the resulting bind value and G12 for the resulting least cost. Enter the following formulas:

1. Total weight: Enter in H12

   $$= SUM(H6 : H10)$$

   and \$H\$12 = 100.



	A	B	C	D	E	F	G	H	I
2									
3			Composition						
4	Meat type	Fat	Protein	Water		Bind Constant	Cost	Weight	
5						($/kg)			
6	Bull meat	11.8	19.1	67.9		30.01	1.8	58.54387	
7	Pork trm	25	15.9	57.4		19.25	1.41	18.86792	
8	Beef chk	14.2	17.3	68		13.96	1.17	2.525621	
9	Pork trm	14.1	17.3	67		8.61	1.54	0	
10	Back fat	89.8	1.9	8.3		1.13	0.22	20.06259	
11								100	
12		30	15			21.78037357	1.393515	100	
13									
14			3						
15									

Figure 3.18 Solution to a least-cost meat formulation problem using the Solver macro in Excel.

2. Constraint 1: 15% protein in 100 kg mix will require 15 kg of protein. Enter 15 in C12. Constraint 1 specifies 20% of the 15 kg protein or 3 kg protein must come from pork trim and pork cheeks. This constraint is represented by the following equation entered in C14:

   $$= \$H\$7^{*}0.159 + \$H\$9^{*}0.173 \quad \text{and} \quad \$C\$14 = 3$$

3. Because Constraint 2 refers to a meat weight (pork cheeks), the equation representing this constraint is entered in H8 as follows: $\$H\$8 < or = 15$
4. Constraint 3, the cumulative bind value of the meat components is entered in F12 as:

   $$= \$H\$6^{*}0.3001 + \$H\$7^{*}0.1925 + \$H\$8^{*}0.1396 + \$H\$9^{*}0.0861 + \$H\$10^{*}0.0113$$

   and E12 $>$ 15.
5. Cumulative fat from the meat components is entered in B12 as the fat balance formula:

   $$= \$H\$6^{*}0.118 + \$H\$7^{*}0.25 + \$H\$8^{*}0.142 + \$H\$9^{*}0.141 + \$H\$10^{*}0.898$$

   and B12 $=$ 30.
6. Cumulative protein from the meat components is entered in C12:

   $$= \$H\$3^{*}0.191 + \$H\$4^{*}0.159 + \$H\$5^{*}0.173 + \$H\$6^{*}0.173 + \$H\$7^{*}0.019$$

   and: C12 $=$ 15.
7. Other constraints are that no acceptable solution will have any meat component with values less than zero: These will be entered in the constraints box as follows:

   H6 $>=$ 0; H7 $>=$ 0; H9 $>=$ 0; H10 $>=$ 0

8. Formula for total cost is entered in G12:

   $$= \$H\$6^{*}0.018 + \$H\$7^{*}0.0141 + \$H\$8^{*}0.0117 + \$H\$9^{*}0.0154 + \$H\$10^{*}0.0022$$

   The value of G12 must be minimized.

Access the Solver by clicking on Tools and selecting Solver. The Solver Parameters box will appear. In the Set Target Cell box enter G12 and in the row Equal To choose Min. In the By Changing Cells box enter H6 : H10. Add all the constraints into the "Subject to the Constraints" box. Click Solve. Results are displayed in the spreadsheet in Fig. 3.18.

The least = cost formulation has the following meats: 58.54 kg bull meat, 18.87 kg pork trimmings, 2.5 kg beef cheeks, 0 kg pork cheeks, and 20.06 kg back fat. The total mass of meats is 100 kg, and the cost of the formulation is 139.35 $/100 kg.

The least-cost formulation will change as the cost of the ingredients change, therefore, processors need to update cost information regularly and determine the current least cost.

3.4 MULTISTAGE PROCESSES

Problems of this type require drawing a process flow diagram and moving the system boundaries for the material balance formulations around parts of the process as needed to the unknown quantities. It is always good to define the basis used in each stage of the calculations. It is possible to change basis as the calculations proceed from an analysis of one subsystem to the next. A "tie material" will also be helpful in relating one part of the process to another.

Example 3.17. The standard of identity for jams and preserves specify that the ratio of fruit to added sugar in the formulation is 45 parts fruit to 55 parts sugar. A jam also must have a soluble content of at least 65% to produce a satisfactory gel. The standard of identity requires soluble solids of at least 65% for fruit preserves from apricot, peach, pear, cranberry, guava, nectarine, plum, gooseberry, figs, quince, and currants. The process of making fruit preserves involves mixing the fruit and sugar in the required ratio, adding pectin, and concentrating the mixture by boiling in a vacuum, steam-jacketed kettle until the soluble solids content is at least 65%. The amount of pectin added is determined by the amount of sugar used in the formulation and by the grade of the pectin (a 100 grade pectin is one that will form a satisfactory gel at a ratio 1 kg pectin to 100 kg sugar).

If the fruit contains 10% soluble solids and 100 grade pectin is used, calculate the weight of fruit, sugar, and pectin necessary to produce 100 kg of fruit preserve. For quality control purposes, soluble solids are those that change the refractive index and can be measured on a refractometer. Thus, only the fruit soluble solids and sugar are considered soluble solids in this context, and not pectin.

Solution:

The process flow diagram is shown in Fig. 3.19. The boundary used for the system on which the material balance is made encloses the whole process.

This problem uses data given in the problem, 100 kg fruit preserve, as a basis. The tie material is the soluble solids because pectin is unidentifiable in the finished product. The designation "fruit" is meaningless in the finished product because the water that evaporates comes from the fruit, and the soluble solids in the fruit mixes with the rest of the system.

Soluble solids balance:

$$0.1(X) + Y = 100(0.65)$$

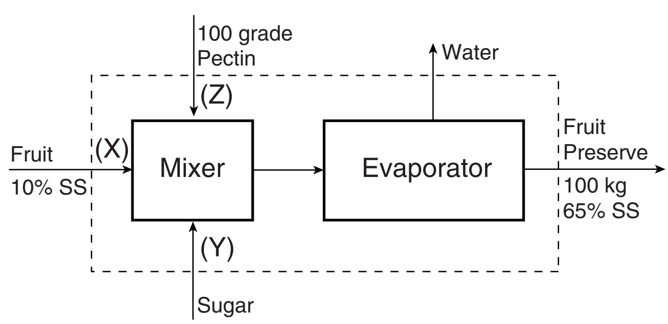


Figure 3.19 Process and material flow for manufacturing fruit preserves.

Because the ratio 45 parts fruit to 55 parts sugar is a requirement, the other equation would be

$$\frac{X}{Y} = \frac{45}{55};\ X = \frac{45}{55}\,Y$$

Substituting the expression for X in the equation representing the soluble solids balance and solving for Y:

$$0.1\left(\frac{45}{55}Y\right) + Y = 65$$

$$0.1(45)(Y) + 55Y = 55(65)$$

$$4.5Y + 55Y = 55(65)$$

$$59.5Y = 55(65)$$

$$Y = \frac{55(65)}{59.5} = 60 \text{ kg sugar}$$

Because $X = 45Y/55$, $X = 45(60)/55 = 49$ kg fruit.

$$Z = \frac{\text{kg sugar}}{\text{grade of pectin}} = \frac{60}{100} = 0.6 \text{ kg pectin}$$

The amount of pectin, Z, can be calculated from the weight of sugar used. The problem can also be solved by selecting a different variable as a basis: 100 kg fruit. Equation based on the required ratio of fruit to sugar:

$$\text{Sugar} = 100 \text{ kg fruit} \times \frac{55 \text{ kg sugar}}{45 \text{ kg fruit}} = 122 \text{ kg}$$

Let X = kg of jam produced.

Soluble solids balance: $100(0.1) + 122 = X(0.65)$

$$X = \frac{132}{0.65} = 203 \text{ kg}$$

Because 100 kg fruit will produce 203 kg jam, the quantity of fruit required to produce 100 kg jam can be determined by ratio and proportion:

$$100 : 203 = X : 100$$

$$X = \frac{100(100)}{203} = 49 \text{ kg fruit}$$

$$\text{Sugar} = 49 \times \frac{55}{45} = 60 \text{ kg}$$

$$\text{Pectin} = \frac{1}{100}(60) = 0.6 \text{ kg}$$

Example 3.18. In solvent extractions, the material to be extracted is thoroughly mixed with a solvent. An ideal system is one where the component to be extracted dissolves in the solvent, and the ratio of solute to solvent in the liquid phase equals the ratio of solute to solvent in the liquid absorbed in the solid phase. This condition occurs with thorough mixing until equilibrium is reached and if sufficient solvent is present such that the solubility of solute in the solvent is not exceeded.

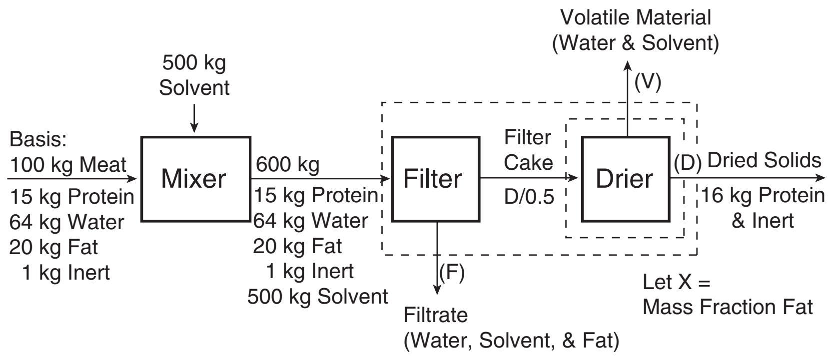


Figure 3.20 Process and material flow for a solvent extraction process involving simultaneous removal of fat and moisture.

Meat (15% protein, 20% fat, 64% water, and 1% inert insoluble solids) is extracted with five times its weight of a fat solvent that is miscible in all proportions with water. At equilibrium, the solvent mixes with water, and fat dissolves in this mixture. Assume there is sufficient solvent such that all the fat dissolves.

After thorough mixing, the solid is separated from the liquid phase by filtration, and the solid phase is dried until all the volatile material is removed. The weight of the dry cake is only 50% of the weight of the cake leaving the filter.

Assume none of the fat, protein, and inerts are removed from the filter cake by drying, and no nonfat solids are in the liquid phase leaving the filter. Calculate the fat content in the dried solids.

Solution:

The diagram of the process is shown in Fig. 3.20. Using 100 kg of meat as a basis, the ratio of solvent to meat of five would require 500 kg of solvent. Consider as a system one that has a boundary that encloses the filter and the drier. Also draw a subsystem with a boundary that encloses only the drier. Write the known weights of process stream and the weights of components in each stream in the diagram. Entering the system at the filter will be 600 kg of material containing 15 kg protein, 1 kg inerts, 64 kg water, 20 kg of fat, and 500 kg of solvent. Because all nonfat solids leave at the drier, the weight of the dried solids D includes 16 kg of the protein and inert material. From the condition given in the problem that the weight of dried solids is 50% of the weight of the filter cake, the material entering the drier should be D/0.5.

All fat dissolves in the solvent-water mixture. Fat, however, will also appear in the solids fraction because some of the fat-water-solvent solution will be retained by the solids after filtration. Although the filtrate does not constitute all of the fat-water-solvent solution, the mass fraction of fat in the filtrate will be the same as in the whole solution, and this condition can be used to calculate the mass fraction of fat in the filtrate.

$$\text{Mass fraction fat in filtrate} = \frac{\text{wt fat}}{\text{wt fat} + \text{wt solv} + \text{wt}\,H_2O}$$

$$= \frac{20}{20 + 64 + 500} = 0.034246$$

Consider the system of the filter and drier. Let x = mass fraction of fat in D. The following component balance can be made.

Fat balance:

$$F(0.034246) + D\,x = 20 \tag{3.1}$$

Protein and inerts balance: Sixteen kilograms of proteins and inerts enter the system and all of these leave the system with the dried solids D. Because D consists only of fat + protein + inert, and x is the mass fraction of fat, then, l − x would be the mass fraction of protein and inert.

$$D(1 - x) = 16; \quad D = \frac{16}{1 - x} \tag{3.2}$$

Solvent and water balance: Five hundred kilograms of solvent and 64 kg of water enter the system. These components leave the system with the volatile material V at the drier and with the filtrate F at the filter. The mass fraction of fat in the filtrate is 0.034246. The mass fraction of water and solvent = 1 − 0.034246 = 0.965754.

$$F(0.965754) + V = 564 \tag{3.3}$$

There are four unknown quantities, F, D, V, and x, and only three equations have been formulated. The fourth equation can be formulated by considering the subsystem of the drier. The condition given in the problem that the weight of solids leaving the drier is 50% of the weight entering would give the following total mass balance equation around the drier:

$$\frac{D}{0.5} = D + V; \qquad D = V \tag{3.4}$$

The above four equations can be solved simultaneously. Substituting D for V in Equation (3.3):

$$F(0.965754) + D = 564 \tag{3.5}$$

Substituting Equation (3.2) in Equation (3.5):

$$F(0.965754) + \frac{16}{1 - x}(x) = 564 \tag{3.6}$$

Substituting Equation (3.2) in Equation (3.1):

$$F(0.034246) + \frac{16}{1x}(x) = 20 \tag{3.7}$$

Solving for F in Equations (3.6) and (3.7) and equating:

$$\frac{20 - 36\,x}{(0.034246)(1 - x)} = \frac{548 - 564x}{(0.965754)(1 - x)}$$

Simplifying and solving for x:

$$\begin{aligned}
0.965754(20 - 36x) &= 0.034246(548 - 564x)\\
19.31508 - 34.767144x &= 18.7668 - 19.314744x\\
x(34.76144 - 19.314744) &= 19.31508 - 18.7668
\end{aligned}$$

$$x = \frac{0.548272}{15.446696} = 0.03549$$

The percentage fat in the dried solids = 3.549%.

The solution can be shortened and considerably simplified if it is realized that the ratio of fat/(solvent + water) is the same in the filtrate as it is in the liquid that adheres to the filter cake entering the drier. Because the volatile material leaving the drier is only solvent + water, it is possible to calculate the amount of fat carried with it from the fat/(solvent + water) ratio.

$$\frac{\text{fat}}{\text{solvent} + \text{water}} = \frac{20}{500 + 64} = 0.03546$$

The amount of fat entering the drier = 0.03546 V. Because D = 16 + fat; D = 16 + 0.03546 V. Total mass balance around the drier gives: D = V.

Substituting D for V gives:

$$\begin{aligned} D &= 16 + 0.03546\ D \\ &= \frac{16}{1 - 0.03546} = 16.5882 \end{aligned}$$

$$\text{wt. fat in D} = 16.5882 - 16 = 0.5882$$

$$\%\ \text{fat in D} = (0.5882/16.5882)(100) = 3.546\%$$

A principle that was presented in an example at the beginning of this chapter was that in solvent extraction or crystallization, the purity of the product depends strongly on the efficiency of separation of the liquid from the solid phase. This is demonstrated here in that the amount of fat entering the drier is a direct function of the amount of volatile matter present in the wet material. Thus, the more efficient the solid-liquid separation process before drying the solids, the less fat will be carried over into the finished product.

Example 3.19. Crystallization. Determine the quantity of sucrose crystals that will crystallize out of 100 kg of a 75% sucrose solution after cooling to 15°C. The mother liquor contains 66% sucrose.

Solution:

The diagram for crystallization is shown in Fig. 3.21. A 75% sucrose solution enters the crystallizer and the material separates into a solution containing 66% sucrose and crystals of 100% sucrose.

Figure 3.21 shows sucrose appearing in the crystals and saturated solution fractions and water appears only in the saturated solution fraction. Water can be used as a tie material in this process.

The mother liquor contains 66% sucrose. The balance (34%) is water. Using the mass fraction principle, 25 kg of water enters the system and all this leaves with the mother liquor where the mass fraction of water is 0.34. Thus, the weight of the mother liquor would be 25/0.34 = 73.52 kg.

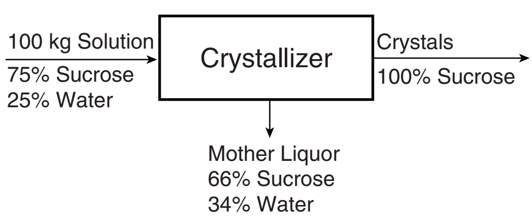


Figure 3.21 Composition and material flow in a crystallization process.

Total mass balance:

wt crystals $= 100 - 73.52 = 26.48$ kg

Example 3.20. For the problem statement and the process represented by the diagram in Fig. 3.2, calculate the yield and the purity of the sugar crystals. The mother liquor contains 67% sucrose w/w. The crystals fraction from the centrifuge loses 15% of its weight in the drier and emerges moisture free.

Solution:

The problem can be separated into two sections. First, a material balance around the evaporator will be used to determine the weight and composition of the material entering the crystallizer. The next part, which involves the crystallizer, centrifuge, and drier, can be solved utilizing the principles demonstrated in the two preceding example problems.

Material balance around the evaporator: Twenty kilograms of sucrose enters, and this represents 75% of the weight of the concentrate.

wt concentrate $= 20/0.75 = 26.66$ kg

The concentrate entering the crystallizer consists of 20 kg sucrose, 1 kg inert, and 5.66 kg water.

Consider the centrifuge as a subsystem where the cooled concentrate is separated into a pure crystals fraction and a mother liquor fraction. The weights of these two fractions and the composition of the saturated solution fraction can be determined using a similar procedure as in the preceding example. In Fig. 3.22, the known quantities indicated on the diagram.

The composition of the mother liquor fraction is determined by making a material balance around the crystallizer. The mass and composition of streams entering and leaving the crystallizer are shown in Fig. 3.23.

All the water and inert material entering the centrifuge go into the mother liquor fraction. The weight of water and inert material in the mother liquor are 5.66 and 1 kg, respectively.

The material balance around the crystallizer is

Total mass: $M + S = 26.66$
Sucrose balance: $M(0.67) + S = 20$

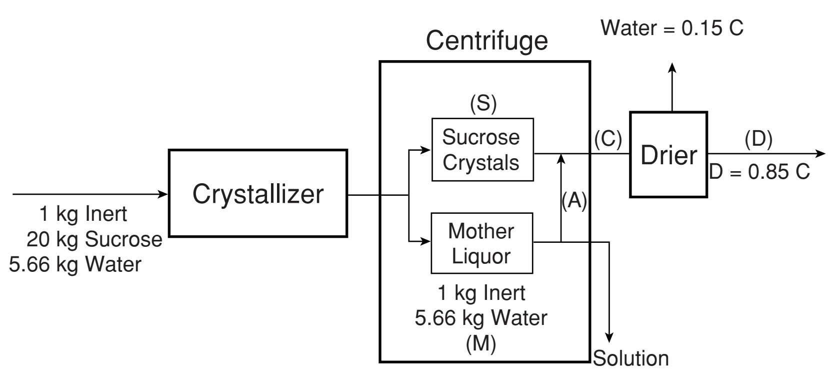


Figure 3.22 Material flow and system boundaries used for analysis of crystal purity from a crystallizer.

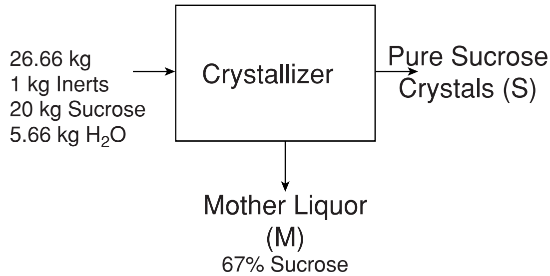


Figure 3.23 Material balance around the crystallizer to determine the quantity of mother liquor and pure crystals that separate on crystallization.

Eliminating S by subtracting the two equations:

$$M(1 - 0.67) = 26.66 - 20; M = 6.66/0.33 = 20.18 \text{ kg}$$

From the known weights of components, the composition of mother liquor, in mass fraction, is as follows.

Sucrose = 0.67, as defined
Inert: 1.00 kg; mass fraction = 1/20.18 = 0.04955
Water: 5.66 kg; mass fraction = 5.66/20.18 = 0.28048

The solids fraction C leaving the centrifuge is a mixture of the mother liquor adhering to the crystals, A, and pure sucrose crystals S. Because 26.66 kg of material entered the crystallizer and 20.18 kg is in the mother liquor, the pure sucrose crystals, S=26.66 − 20.18=6.48 kg.

$$C = 6.48 + A$$

Water balance around the drier:

$$A(\text{mass fraction water in A}) = 0.15\ C$$
$$A(0.28048) = 0.15\ C$$

Substituting C:

$$A(0.28048) = 0.15(6.48 + A)$$
$$A(0.28048 - 0.15) = 0.15(6.48)$$
$$A = 7.4494 \text{ kg}$$
$$C = 6.48 + 7.4494 = 13.9294 \text{ kg}$$

Wt dry sugar, D = 0.85(6.48 + 7.4494) = 11.84 kg
Wt inert material = A(mass fraction inert in A) = 7.4494(0.04955) = 0.3691 kg
% inert in dry sugar = (0.3691/11.84)(100) = 3.117%
The dry sugar is only 96.88% sucrose.

Example 3.21. An ultrafiltration system has a membrane area of 0.75 m^2 and has a water permeability of 180 kg water/h(m^2) under conditions used in concentrating whey from 7% to 25% total solids at a pressure of 1.033 MPa. The system is fed by a pump that delivers 230 kg/h and the appropriate concentration of solids in the product is obtained by recycling some of the product through the membrane. The concentrate contained 11% lactose and the unprocessed when contained 5.3% lactose. There is no protein in the permeate.

Calculate (1) the production rate of 25% concentrate through the system; (2) the amount of product recycled/h; (3) the amount of lactose removed in the permeate/h; (4) the concentration of lactose in the mixture of fresh and recycled whey entering the membrane unit; and (5) the average rejection factor by the membrane for lactose based on the average lactose concentration entering and leaving the unit.

The rejection factor (F_r) of solute through a membrane is defined expressed by:

$$F_r = \frac{X_f - X_p}{X_f}$$

where X_f is solute concentration on the feed side of the membrane, which may be considered as the mean of the solute concentration in the fluid entering and leaving the membrane unit, and X_p is the solute concentration in the permeate.

Solution:

The process flow diagram is shown in Fig. 3.24.

Let L, Pr, and W represent lactose, protein, and water respectively. Let F = feed, P = permeate or fluid passing through the membrane, and R = recycled stream, which has the same composition as the concentrate, C. The fluid stream leaving the membrane unit which is the sum of the concentrate and recycle stream, is also referred to as the retentate.

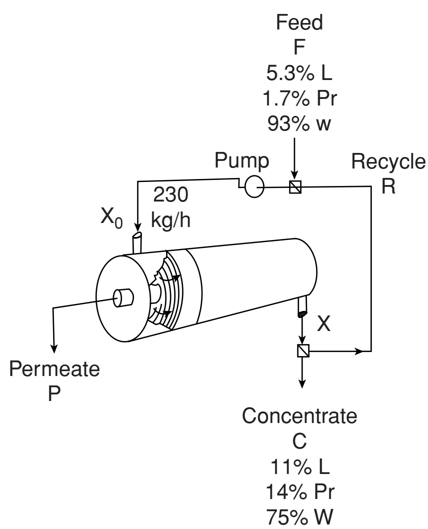


Figure 3.24 Diagram of a spiral wound membrane module in an ultrafiltration system for cheese whey.

Whole system material balance: Basis: 1 hour of operation.

$$\text{Water in permeate} = \frac{180\ \text{kg}}{\text{m}^2\text{h}}(0.75_{\text{m}^2})(1\ \text{h})$$
$$= 180(0.75) = 135\ \text{kg}$$

Water balance:

$$F(0.93) = 135 + C(0.75);\ 0.93F - 0.75C = 135$$

Protein balance:

$$F(0.017) = C(0.14);\ 0.017F - 0.14C = 0$$

Solving for F and C by determinants:

$$\begin{bmatrix} 0.93 & -0.75 \\ 0.017 & -0.14 \end{bmatrix}\begin{bmatrix} F \\ C \end{bmatrix} = \begin{bmatrix} 135 \\ 0 \end{bmatrix}$$

$$F = \frac{135(-0.14) - 0}{0.93(-0.14) - 0.017(-0.75)} = \frac{-18.9}{-0.1302 + 0.01275}$$

$$C = \frac{0 - 0.017(135)}{-0.1302 + 0.01275} = \frac{-2.295}{-0.11745} = 19.54\ \text{kg/h}$$

F = 160.9 kg/h

1. Amount of product = 19.54 kg/h
2. Amount recycled is determined by a material balance around the pump. The diagram is shown in Fig. 3.25.

   $$R = 230 - 160.9 = 69.1\ \text{kg/h}$$

3. Lactose in permeate: Lactose balance around whole system:
   L_p = lactose in permeate = F(0.053) C(0.11)

   $$= 160.9(0.053) - 19.54(0.11) = 6.379$$

   X_p = mass fraction lactose in permeate.

   $$= \frac{6.379}{135 + 6.379} = 0.04511$$

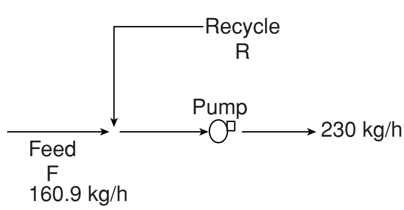


Figure 3.25 Diagram of material balance during mixing of feed and recycle streams in an ultrafiltration system.

4. Lactose balance around pump:

   $F(0.053) + R(0.11) = 230(X_{f1})$

   X_{fi} = mass fraction lactose in fluid leaving the membrane unit.

   $$X_{fl} = \frac{160.9(0.053) + 69.1(0.11)}{230} = 0.0702$$

   X_f = mean mass fraction lactose in the retentate side of the membrane unit.

   $X_f = 0.5(0.0702 + 0.11) = 0.09.$

5. Rejection factor for lactose:

   $F_r = (0.09 - 0.045)/0.09 = 0.498$

PROBLEMS

3.1. A frankfurter formulation is to be made from the following ingredients:

Lean beef: 14% fat, 67% water, 19% protein.
Pork fat: 89% fat, 8% water, 3% protein.
Soy protein isolate: 90% protein, 8% water.

Water needs to be added (usually in the form of ice) to achieve the desired moisture content. The protein isolate added is 3% of the total weight of the mixture. How much lean beef, pork fat, water, and soy isolate must be used to obtain 100 kg of a formulation having the following composition: protein,15%; moisture, 65%; fat, 20%.

3.2. If 100 kg of raw sugar containing 95% sucrose, 3% water, and 2% soluble uncrystallizable inert solids is dissolved in 30 kg of hot water and cooled to 20°C, calculate:

(a) kilograms of sucrose that remains in solution,
(b) crystalline sucrose,
(c) the purity of the sucrose (in % sucrose) obtained after centrifugation and dehydration to 0% moisture. The solid phase contained 20% water after separation from the liquid phase in the centrifuge.

A saturated solution of sucrose at 20°C contains 67% sucrose (w/w).

3.3. Tomato juice flowing through a pipe at the rate of 100 kg/min is salted by adding saturated salt solution (26% salt) into the pipeline at a constant rate. At what rate would the saturated salt solution be added to have 2% salt in the product?

3.4. If fresh apple juice contains 10% solids, what would be the solids content of a concentrate that would yield single-strength juice after diluting one part of the concentrate with three parts of water. Assume densities are constant and are equal to the density of water.

3.5. In a dehydration process, the product, which is at 80% moisture, initially has lost half its weight during the process. What is the final moisture content?

3.6. Calculate the quantity of dry air that must be introduced into an air drier that dries 100 kg/h of food from 80% moisture to 5% moisture. Air enters with a moisture content of 0.002 kg water per kg of dry air and leaves with a moisture content of 0.2 kg H_2O per kg of dry air.

3.7. How much water is required to raise the moisture content of 100 kg of a material from 30% to 75%?

3.8. In the section "Multistage Process," Example 3.18, solve the problem if the meat to solvent ratio is 1:1. The solubility of fat in the watersolvent mixture is such that the maximum fat content in the solution is 10%.

3.9. How many kilograms of peaches would be required to produce 100 kg of peach preserves? The standard formula of 45 parts fruit to 55 parts sugar is used, the soluble solids content of the finished product is 65% and the peaches have 12% initial soluble solids content. Calculate the weight of 100 grade pectin required and the amount of water removed by evaporation.

3.10. The peaches in Problem 9 come in a frozen form to which sugar has been added in the ratio of three parts fruit to one part sugar. How much peach preserves can be produced from 100 kg of this frozen raw material?

3.11. Yeast has a proximate analysis of 47% C, 6.5% H, 31% O, 7.5% N, and 8% ash on a dry weight basis. Based on a factor of 6.25 for converting protein nitrogen to protein, the protein content of yeast on a dry basis is 46.9%. In a typical yeast culture process, the growth medium is aerated to convert substrate primarily to cell mass. The dry cell mass yield is 50% of a sugar substrate. Nitrogen is supplied as ammonium phosphate.

The cowpea is a high-protein, low-fat legume that is a valuable protein source in the diet of many Third World nations. The proximate analysis of the legume is 30% protein, 50% starch, 6% oligosaccharides, 6% fat, 2% fiber, 5% water, and 1% ash. It is desired to produce a protein concentrate by fermenting the legume with yeast. Inorganic ammonium phosphate is added to provide the nitrogen source. The starch in cowpea is first hydrolyzed with amylase and yeast is grown on the hydrolyzate.

(a) Calculate the amount of added inorganic nitrogen as ammonium phosphate to provide the stoichiometric amount of nitrogen to convert all the starch present to yeast mass. Assume none of the cowpea protein is utilized by the yeast.

(b) If the starch is 80% converted to cell mass, calculate the proximate analysis of the fermented cowpea on a dry basis.

3.12. This whey is spray dried to a final moisture content of 3%, and the dry whey is used in an experimental batch of summer sausage.

In summer sausage, the chopped meat is inoculated with a bacterial culture that converts sugars to lactic acid as the meat is allowed to ferment prior to cooking in a smokehouse. The level of acid produced is controlled by the amount of sugar in the formulation. The lactic acid level in the summer sausage is 0.5 g/100 g dry solids. Four moles of lactic acid ($CH_3CHOHCOOH$) is produced from 1 mole of lactose ($C_{12}H_{22}O_{11}$). The following formula is used for the summer sausage:

3.18 kg lean beef (16% fat, 16% protein, 67.1% water, 0.9% ash)
1.36 kg pork (25% fat, 12% protein, 62.4% water, 0.6% ash)
0.91 kg ice
0.18 kg soy protein isolate (5% water, 1% ash, 94% protein)

Calculate the amount of dried whey protein that can be added into this formulation in order that, when the lactose is 80% converted to lactic acid, the desired acidity will be obtained.

3.13. Osmotic dehydration of blueberries was accomplished by contacting the berries with a corn syrup solution containing 60% soluble solids for 6 hours and draining the syrup from the solids. The solid fraction left on the screen after draining the syrup is 90% of the original weight of the berries. The berries originally contained 12% soluble solids, 86.5% water, and 1.5% insoluble solids. The sugar in the syrup penetrated the berries such that the berries themselves remaining on the screen, when washed free of the adhering solution, showed a soluble solids gain of 1.5% based on the original dry solids content. Calculate:

(a) The moisture content of the berries and adhering solution remaining on the screen after draining the syrup.
(b) The soluble solids content of the berries after drying to a final moisture content of 10%.
(c) The percentage of soluble solids in the syrup drained from the mixture. Assume none of the insoluble solids are lost in the syrup.

3.14. The process for producing dried mashed potato flakes involves mixing wet mashed potatoes with dried flakes in a 95:5 weight ratio, and the mixture is passed through a granulator before drying on a drum dryer. The cooked potatoes after mashing contained 82% water and the dried flakes contained 3% water. Calculate:

(a) The amount of water that must be removed by the dryer for every 100 kg of dried flakes produced.
(b) The moisture content of the granulated paste fed to the dryer.
(c) The amount of raw potatoes needed to produce 100 kg of dried flakes; 8.5% of the raw potato weight is lost on peeling.
(d) Potatoes should be purchased on a dry matter basis. If the base moisture content is 82% and potatoes at this moisture content cost $200/ton, what would be the purchase price for potatoes containing 85% moisture.

3.15. Diafiltration is a process used to reduce the lactose content of whey recovered using an ultrafiltration membrane. The whey is passed through the membrane first and concentrated to twice the initial solids content, rediluted, and passed through the membrane a second time. Two membrane modules in series, each with a membrane surface area of 0.5 m^2, are to be used for concentrating and removing lactose from acid whey that contains 7.01% total solids, 5.32% lactose, and 1.69% protein. The first module accomplishes the initial concentration, and the retentate is diluted with water and reconcentrated in the second module to a solids content of 14.02%. Under the conditions used in the process, the membrane has an average water permeation rate of 254 kg/(h $\cong$ m^2). The rejection factor for lactose by the membrane based on the arithmetic mean of the feed and retentate lactose concentration and the mean permeate lactose concentration is 0.2. Protein rejection factor is 1. The rejection factor is defined as: $F_r = (C_r C_p)/C_r$ where C_r is the concentration on the retentate side and C_p is the concentration on the permeate side of the membrane. Use Visual BASIC to determine:

(a) The amount of 14.02% solids delactosed whey concentrate produced from the second module per hour.
(b) The lactose content of the delactosed whey concentrate.

3.16. An orange juice blend containing 42% soluble solids is to be produced by blending stored orange juice concentrate with the current crop of freshly squeezed juice. The following are the constraints:

The soluble solids to acid ratio must equal 18; and the currently produced juice may be concentrated first before blending, if necessary. The currently produced juice contains 14.5% soluble solids, 15.3% total solids, and 0.72% acid. The stored concentrate contains 60% soluble solids, 62% total solids, and 4.3% acid. Calculate:

(a) The amount of water that must be removed or added to adjust the concentration of the soluble solids in order to meet the specified constraints.

(b) The amounts of currently processed juice and stored concentrate needed to produce 100 kg of the blend containing 42% soluble solids.

3.17. The process for extraction of sorghum juice from sweet sorghum for production of sorghum molasses, which is still practiced in some areas in the rural South of the United States, involves passing the cane through a three-roll mill to squeeze the juice out. Under the best conditions, the squeezed cane (bagasse) still contains 50% water.

(a) If the cane originally contained 13.4% sugar, 65.6% water, and 32% fiber, calculate the amount of juice squeezed from the cane per 100 kg of raw cane, the concentration of sugar in the juice, and the percentage of sugar originally in the cane that is left unrecovered in the bagasse.

(b) If the cane is not immediately processed after cutting, moisture and sugar loss occurs. Loss of sugar has been estimated to be as much as 1.5% within a 24-hour holding period, and total weight loss for the cane during this period is 5.5%. Assume sugar is lost by conversion to CO_2, therefore the weight loss is attributable to water and sugar loss. Calculate the juice yield based on the freshly harvested cane weight of 100 kg, the sugar content in the juice, and the amount of sugar remaining in the bagasse.

3.18. In a continuous fermentation process for ethanol from a sugar substrate, the sugar is converted to ethanol and part of it is converted to yeast cell mass. Consider a 1000 L continuous fermentor operating at a steady state. Cell-free substrate containing 12% glucose enters the fermentor. The yeast has a generation time of 1.5 hours, and the concentration of yeast cells within the fermentor is 1×10^7 cells/mL. Under these conditions, a dilution rate (F/V, where F is the rate of feeding of cell-free substrate and V is the volume of the fermentor) that causes the cell mass to stabilize at a steady state results in a residual sugar content in the overflow of 1.2%. The stoichiometric ratio for sugar to dry cell mass is 1:0.5 on a weight basis, and that for sugar to ethanol is based on 2 moles of ethanol produced per mole of glucose. A dry cell mass of 4.5 g/L is equivalent to a cell count of 1.6×10^7 cells/mL. Calculate the ethanol productivity of the fermentor in g ethanol/(L $\cong$ h).

3.19. A protein solution is to be dimineralized by dialysis. The solution is placed inside a dialysis tube immersed in continuously flowing water. For all practical purposes, the concentration of salt in the water is zero, and the dialysis rate is proportional to the concentration of salt in the solution inside the tube. The contents of the tube may be considered to be well mixed. A solution that initially contained 500 μg/mL of salt and 15 mg protein/mL contained 400 μg/mL salt and 13 mg protein/mL at the end of 2 hours. Assume no permeation of protein through the membrane, and density of the solution is constant at 0.998 g/mL. The rate of permeation of water into the membrane and the rate of permeation of salt out of the membrane are both directly proportional to the concentration of salt in the solution inside the membrane. Calculate the time of dialysis needed to drop the salt concentration inside the membrane to 10 μg/mL. What will be the protein concentration inside the membrane at this time?

3.20. A fruit juice–based natural sweetener for beverages is to be formulated. The sweetener is to have a soluble solids to acid ratio of 80. Pear concentrate having a soluble solids to acid ratio of 52 and osmotically concentrated/deacidified apple juice with a soluble solids to acid ratio of 90 are available. Both concentrates contain 70% soluble solids.

(a) Calculate the ratio of deacidified apple and pear concentrates that must be mixed to obtain the desired sol. solids/acid ratio in the product.

(b) If the mixture is to be diluted to 45% soluble solids, calculate the amounts of deacidified apple and pear juice concentrates needed to make 100 kg of the desired product.

3.21. A spray drier used to dry egg whites produces 1000 kg/h of dried product containing 3.5% moisture from a raw material that contains 86% moisture.
(a) If air used for drying enters with a humidity of 0.0005 kg water per kg dry air and leaves the drier with a humidity of 0.04 kg water per kg dry air, calculate the amount of drying air needed to carry out the process.
(b) It is proposed to install a reverse osmosis system to remove some moisture from the egg whites prior to dehydration to increase the drying capacity. If the reverse osmosis system changes the moisture content of the egg whites prior to drying to 80% moisture, and the same inlet and outlet air humidities are used on the drier, calculate the new production rate for the dried egg whites.

3.22. An ultrafiltration system for concentrating milk has a membrane area of 0.5 m^2, and the permeation rate for water and low molecular weight solutes through the membrane is 3000 g/($m^2 \cong$ min). The solids content of the permeate is 0.5%. Milk flow through the membrane system must be maintained at 10 kg/min to prevent fouling. If milk containing initially 91% water enters the system, and a concentrate containing 81% solids is desired, calculate:
(a) The amount of concentrate produced by the unit/h.
(b) The fraction of total product leaving the membrane that must be recycled to achieve the desired solids concentration in the product.
(c) The solids concentration of feed entering the unit after the fresh and recycled milk are mixed.

3.23. A recent development in the drying of bluberries involves an osmotic dewatering prior to final dehydration. In a typical process, grape juice concentrate with 45% soluble solids (1.2% insoluble solids; 53.8% water) in a ratio 2 kg juice/kg berries is used to osmoblanch the berries, followed by draining the berries and final drying in a tunnel drier to a moisture content of 12%.

During osmoblanching (the juice concentrate is heated to 80°C, the berries added, temperature allowed to go back to 80°C, 5-minute hold, and the juice is drained), the solids content of blueberies increases. When analyzed after rinsing the adhering solution, the total solids content of the berries was 15%. The original berries contained 10% total solids and 9.6% soluble solids. Assume leaching of blueberry solids into the grape juice is negligible.
The drained berries carry about 12% of their weight of the solution that was drained.
Calculate:
(a) The proximate composition of the juice drained from the blueberries.
(b) The yield of dried blueberries from 100 kg raw berries.
(c) The drained juice is concentrated and recycled. Calculate the amount of 45% solids concentrate that must be added to the recycled concentrate to make enough for the next batch of 100 kg blueberies.

3.24. A dietetic jelly is to be produced. In order that a similar fruit flavor may be obtained in the dietetic jelly as in the traditional jelly, the amount of jelly that can be made from a given amount of fruit juice should be the same as in the standard pectin jelly.

If 100 kg of fruit juice is available with a soluble solids content of 14%, calculate the amount of standard pectin jelly with a soluble solids content of 65% that can be produced.

Only the sugar in jelly contribute the caloric content.
(a) What will be the caloric equivalent of 20 g (1 tsp) of standard pectin jelly?
(b) The dietetic jelly should have a caloric content 20% that of the standard pectin jelly. Fructose may be added to provide sweetness. The low methoxy pectin can be used in the

same amount for the same quantity of fruit as in the standard jelly. Calculate the soluble solids content in the dietetic jelly to give the caloric reduction.

(c) In part (b), calculate the amount of additional fructose that must be used.

SUGGESTED READING

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